p-BRANE ACTION FROM GRAVI-DILATON EFFECTIVE ACTION

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Abstract

Using a special ansatz for the metric, by straightforward computation we prove that gravi-dilaton effective action in higher dimensions is reduced to the p-brane action. The dual symmetry of the generic type $a \longleftrightarrow \frac{1}{a}$ is an important symmetry of the reduced action.

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In this brief note, by a straightforward computation we prove the surprising result that using a particular ansatz for the metric the gravi-dilaton action in $d+1$ dimensions is reduced to the $p-$brane action. This result may be of special interest in the Randall-Sundrum scenario [1-2], string theory [3] and M-theory [4].

Our starting point is the graviton-dilaton effective action with cosmological constant:

$$S = -\tfrac{1}{16\pi G_{d+1}} \int d^{d+1}y\sqrt{-g}e^{-\phi}(R + (\nabla\phi)^2 + 2\Lambda), \tag{1}$$

where G_{d+1} is the Newton constant in $d+1$ dimensions, $\phi = \phi(y^\alpha)$ is the dilaton field and R is the Ricci scalar obtained from the Riemann tensor

$$R^\mu_{\nu\alpha\beta} = \Gamma^\mu_{\nu\beta,\alpha} - \Gamma^\mu_{\nu\alpha,\beta} + \Gamma^\mu_{\sigma\alpha}\Gamma^\sigma_{\nu\beta} - \Gamma^\mu_{\sigma\beta}\Gamma^\sigma_{\nu\alpha} \tag{2}$$

and the metric tensor $g_{\alpha\beta}$, with $\alpha, \beta = 0, 1..., d$. Here, $\Gamma^\mu_{\alpha\beta}$ is the Christoffel symbol:

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\nu}(g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu}). \tag{3}$$

Consider the ansatz

$$\begin{aligned} g_{AB} &= \tilde{g}_{AB}(y^C), \\ g_{ij} &= a_k(y^C)a_l(y^C)\eta^{kl}_{ij}. \\ g_{Ai} &= 0. \end{aligned} \tag{4}$$

Here, the indices $A, B, ...etc.$ run from 0 to p, the indices $i, j, ...etc.$ run from $p+1$ to d and the only non-vanishing terms of η^{kl}_{ij} are

$$\eta^{kl}_{ij} = 1, \text{ when } k = l = i = j. \tag{5}$$

Assume that

$$\phi = \phi(y^C). \tag{6}$$

From (3), (4) and (6) we find that the only non-vanishing Christoffel symbols are

$$
\begin{aligned}
\Gamma^A_{ij} &= -a_k \partial^A a_l \eta^{kl}_{ij}, \\
\Gamma^i_{jA} &= a^k \partial_A a_l \eta^{li}_{kj}, \qquad (7) \\
\Gamma^A_{BC} &= \tilde{\Gamma}^A_{BC},
\end{aligned}
$$

where $\tilde{\Gamma}^A_{BC}$ is the Christoffel symbol associated to $\tilde{g}_{AB}$. Here, $a^i = a_i^{-1}$, so we can take $g^{ij} = a^k a^l \eta^{ij}_{kl}$.

From (2), (4) and (7) we discover that the only non-vanishing components of the Riemann tensor are

$$
\begin{aligned}
R^A_{BCD} &= \tilde{R}^A_{BCD}, \\
R^A_{iBj} &= -a_k D_B \partial^A a_l \eta^{kl}_{ij}, \\
R^i_{AjB} &= -a^k D_B \partial_A a_l \eta^{li}_{kj}, \qquad (8) \\
R^i_{jkl} &= -a^m \partial^A a_n a_r \partial_A a_s (\eta^{ni}_{mk} \eta^{rs}_{jl} - \eta^{ni}_{ml} \eta^{rs}_{jk}).
\end{aligned}
$$

where D_A denotes a covariant derivative in terms of $\tilde{\Gamma}^A_{BC}$. From (8) we find that the non-vanishing components of the Ricci tensor $R_{\mu\nu} \equiv R^\alpha_{\mu\alpha\nu}$ are

$$
\begin{aligned}
R_{AB} &= \tilde{R}_{AB} - a^i D_B \partial_A a_i, \qquad (9) \\
R_{ij} &= -(a_k D^A \partial_A a_l + a^m a_k \partial_A a_m \partial^A a_l - \partial_A a_k \partial^A a_l) \eta^{kl}_{ij}.
\end{aligned}
$$

Thus, the Ricci scalar $R = g^{\mu\nu} R_{\mu\nu}$ is given by

$$
\begin{aligned}
R &= -2a^i D^A \partial_A a_i - a^i a^j \partial_A a_i \partial^A a_j \qquad (10) \\
&\quad + a^i a^j \partial_A a_k \partial^A a_l \eta^{kl}_{ij} + \tilde{R}.
\end{aligned}
$$

Therefore, the action (1) becomes

$$S = -\tfrac{1}{16\pi G_{p+1}}\int d^{p+1}y\sqrt{-\tilde{g}}\Pi a_s e^{-\phi}\{-2a^i D^A\partial_A a_i - a^i a^j\partial^A a_i\partial_A a_j$$

$$+a^i a^j\partial^A a_k\partial_A a_l\eta_{ij}^{kl} + \partial^A\phi\partial_A\phi + \tilde{R} + 2\Lambda\}, \tag{11}$$

where G_{p+1} is the Newton constant in $p+1$ dimensions. The relation between G_{p+1} and G_{d+1} is

$$\frac{1}{G_{p+1}} = \frac{V_n}{G_{d+1}}, \tag{12}$$

where V_n is a volume element in $n = d - p$ dimensions. This action can be rewritten as

$$S = -\tfrac{1}{16\pi G_{p+1}}\int d^{p+1}x\sqrt{-\tilde{g}}D^A(-2\Pi a_s e^{-\phi}a^i\partial_A a_i)$$

$$-\tfrac{1}{16\pi G_{p+1}}\int d^{p+1}y\sqrt{-\tilde{g}}\Pi a_s e^{-\phi}\{a^i a^j\partial^A a_i\partial_A a_j - 2\partial^A\phi a^i\partial_A a_i \tag{13}$$

$$+\partial^A\phi\partial_A\phi - a^i a^j\partial^A a_k\partial_A a_l\eta_{ij}^{kl} + 2\Lambda\} - \tfrac{1}{16\pi G_{p+1}}\int d^{p+1}y\sqrt{-\tilde{g}}\Pi a_s e^{-\phi}\tilde{R}.$$

Classically, since the first term in (13) is a total derivative we can be drop it. Therefore, we have

$$S = -\tfrac{1}{16\pi G_{p+1}}\int d^{p+1}y\sqrt{-\tilde{g}}\Pi a_s e^{-\phi}\{a^i a^j\partial^A a_i\partial_A a_j - 2\partial^A\phi a^i\partial_A a_i +$$

$$+\partial^A\phi\partial_A\phi - a^i a^j\partial^A a_k\partial_A a_l\eta_{ij}^{kl} + 2\Lambda\} - \tfrac{1}{16\pi G_{p+1}}\int d^{p+1}y\sqrt{-\tilde{g}}\Pi a_s e^{-\phi}\tilde{R}. \tag{14}$$

Let us define x^0 as

$$\Pi a_s e^{-\phi} = e^{-x^0}. \tag{15}$$

We find

$$x^0 = \phi - \sum \ln a_s \tag{16}$$

and therefore,

$$\partial_A x^0 = \partial_A \phi - a^s \partial_A a_s. \tag{17}$$

Thus, the action (13) becomes

$$S = \frac{1}{16\pi G_{p+1}} \int d^{p+1}y \sqrt{-\tilde{g}} e^{-x^0} \{-\partial^A x^0 \partial_A x^0 + a^i a^j \partial^A a_k \partial_A a_l \eta_{ij}^{kl} - 2\Lambda\}$$
$$- \frac{1}{16\pi G_{p+1}} \int d^{p+1}y \sqrt{-\tilde{g}} e^{-x^0} \tilde{R}, \tag{18}$$

It is not difficult to see that (18) is invariant under the duality transformation

$$a_i \longleftrightarrow \frac{1}{a_i}.$$

Let us define the p-brane coupling "constant" Ω_p in the form

$$\frac{e^{-x^0}}{16\pi G_{p+1}} = \frac{1}{2\Omega_p} \tag{19}$$

and the variables x^i as

$$x^i \equiv \ln a_i. \tag{20}$$

Using the expression (19) we find that (18) can be rewritten in the form

$$S = \frac{1}{2} \int \frac{d^{p+1}y}{\Omega_p} \sqrt{-\tilde{g}} \{-\partial^A x^0 \partial_A x^0 + a^i a^j \partial^A a_k \partial_A a_l \eta_{ij}^{kl} - 2\Lambda\}$$
$$- \frac{1}{2} \int \frac{d^{p+1}y}{\Omega_p} \sqrt{-\tilde{g}} \tilde{R}. \tag{21}$$

If we now consider the definition (20) we find that (21) becomes

$$S = \frac{1}{2} \int \frac{d^{p+1}y}{\Omega_p} \sqrt{-\tilde{g}} \{-\partial^A x^0 \partial_A x^0 + \partial^A x^i \partial_A x^j \delta_{ij} - 2\Lambda\}\}$$
$$- \frac{1}{2} \int \frac{d^{p+1}y}{\Omega_p} \sqrt{-\tilde{g}} \tilde{R}. \tag{22}$$

We easily note that (22) can be written as

$$S = \frac{1}{2} \int \frac{d^{p+1}y}{\Omega_p} \sqrt{-\tilde{g}} [\tilde{g}^{AB} \partial_A x^{\hat{\mu}} \partial_B x^{\hat{\nu}} \eta_{\hat{\mu}\hat{\nu}} - 2\Lambda]$$
$$- \frac{1}{2} \int \frac{d^{p+1}y}{\Omega_p} \sqrt{-\tilde{g}} \tilde{R}, \tag{23}$$

where $\eta_{\hat{\mu}\hat{\nu}} = diag(-1, 1, ..., 1)$. Here the indices $\hat{\mu}, \hat{\nu}$, ...etc run from 0 to $n = d-p$. By setting the cosmological constant Λ as

$$\Lambda = \frac{p-1}{2}, \tag{24}$$

we recognize the action (23) as the p-brane action.

Let us make some final comments. We have shown explicitly that the p-brane structure is contained in a higher dimensional effective gravi-dilaton theory. In fact, our result is very general since applies to any higher dimensional effective gravi-dilaton theory and any p-brane. The case of 0-brane (point particle) corresponds to a cosmological model and in fact such a case has already been considered in the literature (see Ref. [5] and references there in). The case of 1-brane, corresponding to strings, is of great importance and deserve a special discussion.

From the point of view of the traditional string theory history our result is clearly intriguing and surprising. Since for strings $p = 1$, from (24) we see that $\Lambda = 0$ and the action (23) is reduced to the well-known Polyakov action. Let us assume that this reduced action is the bosonic sector of the superstring action. We know that for superstrings, at the quantum level, the Weyl invariance implies that $n+1 = 10$. Therefore, in this case the effective action (1) is defined in $d + 1 = 11$ dimensions. Consequently, (1) can be thought as the bosonic sector of eleven dimensional supergravity and in this sense our result is in agreement with M-theory. However, let us play with history and assume that the proof of the present work appears before the concept of strings or p-branes emerged in the literature. Clearly, in this sense we can even think that eleven dimensional supergravity is M-theory itself!

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